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1995 J. Phys.: Condens. Matter 7 175

(http://iopscience.iop.org/0953-8984/7/1/015)

J. Phys.: Condens. Matter 7 (1995) 175–184. Printed in the UK

The electromagnetic autowaves in a weakly conductive easy-axis ferromagnet

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Received 9 September 1993, in final form 12 August 1994

Abstract. Propagation of picosecond electromagnetic video pulses along the easy magnetization axis of a non-equilibrium weakly conductive ferromagnet located in an external magnetic field parallel to this axis has been studied. For magnetic field components, solutions in the form of two types of autowave dissipative structure have been obtained. It is shown that these autowave dissipative structures can be formed under the restrictions on the magnetic anisotropy parameter and the Zeeman splitting frequency.

1. Introduction

Recently it has become possible to generate electromagnetic video pulses, i.e. pulses containing one period of oscillation (Auston *et al* 1984, Fork *et al* 1987, Darrow *et al* 1990). Besides femtosecond pulses (Auston *et al* 1984, Fork *et al* 1987), the so-called 'infrared' video pulses of picosecond duration have been generated (Darrow *et al* 1990). In this connection, the study of the non-resonant interaction of such pulses with matter is of considerable interest. In the theoretical papers of Belenov *et al* (1988, 1991, 1992), Belenov and Nazarkin (1990), Maimistov and Elyutin (1991), Sazonov (1991, 1992), Azarenkov *et al* (1991), Sazonov and Yakupova (1992, 1994), Dubrovskaya and Sukhorukov (1992) and Sazonov and Trifonov (1994) the propagation of femtosecond light pulses in two-level non-resonant media has been studied. The papers of Nakata (1991a–c), Sazonov (1993) and Sazonov and Trifonov (1993) were dedicated to the interaction of picosecond video pulses in an isotropic dielectric ferromagnet and paramagnet located in an external magnetic field $\boldsymbol{H}_0$. In particular, Nakata (1991c) enquires into the propagation of a weakly non-linear circularly polarized electromagnetic pulse along the magnetic field $\boldsymbol{H}_0$. At this point, the propagation is described by the 'derivative non-linear Schrödinger (DNLS) equation' for transverse components of ferromagnetic magnetization vector $\boldsymbol{M}$. It is a matter of common knowledge that this equation is integrable by the inverse scattering transform method.

However, many ferromagnets comprise not only localized magnetic moments but conduction electrons. These electrons can interact effectively with a pulsed electric field. Absorbing pulse energy, on collision with the atoms of the lattice, electrons inevitably lose energy. Therefore, electrical resistance and appropriate energy losses take place.

The present paper is concerned with electromagnetic pulse propagation in a ferromagnet subject to electron conductivity.

In sections 2 and 3 we derive the coupled set of equations (16), (18) and (19) that describe the dynamics of the interaction between the magnetization of the ferromagnet and the pulsed magnetic field. By this means the set of equations is the 'jumping-off point' for the main investigation. Two types of approximate solution of these equations as autowave dissipative structures are obtained in section 4. It is shown that these autowaves can be formed under any restrictions on the magnetic anisotropy parameter and the Zeeman splitting frequency. This point is the most significant result of the present paper. The term 'autowave' is a blanket one in the physics, biology and chemistry of non-equilibrium systems (see e.g. Krinsky 1984, Sazonov 1990). An autowave propagating in a non-equilibrium medium takes the energy accumulated in this medium. The gain process is saturated commonly by some dissipative mechanism (e.g. by diffusion) and by non-linearity. Autowave solutions are non-integrable non-linear model ones. Autowaves interact in a non-elastic manner with similar ones. For example, a collision of two autowaves with amplitudes of opposite signs will yield a mutual annihilation. In section 4 we show also that basically the formation of the first-type autowave is caused by the spin-wave interaction, whereas the second-type autowave can be formed due to anisotropic (spin–spin or spin–orbit) interactions. Thus we finally arrive at the conclusion that only the autowave of the second type can be realized experimentally in typical parameters of the medium.

2. Basic model

Let us direct the z axis along the external magnetic field $\boldsymbol{H}_0$ and investigate the propagation of an electromagnetic pulse in a ferromagnet along this axis. The Maxwell equations

$$\nabla \times \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} \tag{1}$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial}{\partial t}(\boldsymbol{H} + 4\pi \boldsymbol{M}) \tag{2}$$

$$\nabla \cdot (\boldsymbol{H} + 4\pi \boldsymbol{M}) = 0 \tag{3}$$

will hold.

Here $\boldsymbol{H}$ and $\boldsymbol{E}$ are respectively the magnetic and electric components of the pulsed field, c is the speed of light and $\boldsymbol{j}$ is the electric current density caused by conduction electrons. If the average time of free electron propagation satisfies the condition $\tau_e \ll \tau$, then Ohm's law is valid:

$$\boldsymbol{j} = \sigma \boldsymbol{E} \tag{4}$$

where σ is the electrical conductivity of the ferromagnet. For simplicity, the electron conductivity will be considered as a scalar value (i.e. the electrical properties of the ferromagnet are considered to be isotropic). Also, we fully ignore the electric medium polarization and assume $\boldsymbol{D} = \boldsymbol{E}$, where $\boldsymbol{D}$ is the electric induction vector. This approximation is justifiable under the condition (Sazonov 1993)

$$(\boldsymbol{d}_j \cdot \boldsymbol{E}/\hbar\omega_j)^2 \ll 1 \tag{5}$$

where $\boldsymbol{d}_j$ is the dipole moment of the electric dipole transition from the quantum level under consideration to one of the nearest quantum levels, and ω_j is the corresponding transition frequency.

The dynamics of the magnetization vector $\boldsymbol{M}$ is described by the Landau–Lifshitz equation (Kosevich *et al* 1985):

$$\partial \boldsymbol{M}/\partial t = (2\beta_0/\hbar)\boldsymbol{M} \times \boldsymbol{H}_{\text{eff}} \tag{6}$$

where the effective magnetic field H_{eff} is defined by the relationship:

$$\boldsymbol{H}_{\text{eff}} = -\partial W/\partial \boldsymbol{M}. \tag{7}$$

Here W is the energy functional of the ferromagnet, determined by the following expression (Kosevich *et al* 1985):

$$W = \int [\tfrac{1}{2}\alpha(\nabla \boldsymbol{M})^2 - \tfrac{1}{2}\beta M_z^2 - (\boldsymbol{H} + \boldsymbol{H}_0)\cdot \boldsymbol{M}]\, \mathrm{d}^3\boldsymbol{r} \tag{8}$$

where α is the exchange interaction constant and β is the anisotropy constant. In the case of an easy-axis-type ferromagnet, $\beta > 0$, whereas $\beta < 0$ in the case of an easy-plane-type ferromagnet.

We assume that the direction of the magnetic field $\boldsymbol{H}_0$ coincides with the direction of the easy magnetization axis.

Let us estimate the terms of (8). By the order of magnitude (Akhiezer *et al* 1967), $\alpha \sim k_B T_K h^5/\beta_0^2$, where k_B is the Boltzmann constant, T_K is the temperature of the ferromagnetic phase transition and h is the distance between the nearest neighbours in the crystal. Then

$$\tfrac{1}{2}\alpha(\nabla \boldsymbol{M})^2 \sim k_B T_K/hc^2\tau^2.$$

Substituting here $T_K \sim 10^2$ K, $h \sim 10^{-8}$ cm and $\tau \sim 10^{-12}$ s, we have $\frac{1}{2}\alpha(\nabla \boldsymbol{M})^2 \sim 10^{-3}$ erg cm^{-3}. For the second term (Akhiezer *et al* 1967)

$$\tfrac{1}{2}\beta M_z^2 \sim (v_e/c)^2(e^2/a_0 h^3)$$

where v_e is the velocity of an atomic electron, a_0 is the Bohr radius and e is the electronic charge. Substituting $(v_e/c)^2 \sim 10^{-5}$, we yield that $\frac{1}{2}\beta M_z^2 \sim 10^9$ erg cm^{-3}.

For the last term in (8), we estimate $\boldsymbol{H}\cdot\boldsymbol{M} \sim \beta_0 H/h^3$. The value τ is defined by the Rabi frequency $\beta_0 H/\hbar$. Consequently, $\beta_0 H \sim \hbar\tau^{-1}$. Then $\boldsymbol{H}\cdot\boldsymbol{M} \sim \hbar/(\tau h^3) \sim 10^9$ erg cm^{-3}.

Thus, the first term (the energy of the exchange interactions) in (8) can be ignored. One may show that the inequality $\alpha|\nabla \boldsymbol{M}|^2 \ll |\boldsymbol{H}\cdot\boldsymbol{M}|$ is identical to $v_m \ll c$, where v_m is the velocity of the free magnons in a ferromagnet. In fact, linearizing (6)–(8), we obtain the dispersion relationship $\omega(k)$ for the spin wave:

$$\omega = (k_B T_K/\hbar)h^2k^2.$$

So,

$$\frac{v_m}{c} \sim \frac{k_B T_K}{\hbar c}h^2 k \sim \frac{k_B T_K}{\hbar}\frac{h^2}{c^2\tau} \sim \frac{\alpha|\nabla \boldsymbol{M}|^2}{|\boldsymbol{H}\cdot\boldsymbol{M}|}.$$

Summarizing the preceding, from (7) and (8) we have

$$\boldsymbol{H}_{\text{eff}} = (H_0 + \beta M_z)e_z + \boldsymbol{H} \tag{9}$$

where e_z is the unit vector parallel to the z axis.

3. The coupled set of non-linear equations: effective transverse pulsed magnetic field

Suppose that an electromagnetic pulse propagates along the z axis and its dynamics is defined by the variables z and t. Then the first equation (3) is easily integrated:

$$H_z + 4\pi M_z = f_1(t) \tag{10}$$

where $f_1(t)$ is a time function.

From (1), (2) and (4) we obtain

$$\nabla \cdot (\nabla \cdot \boldsymbol{H}) - \Delta \boldsymbol{H} = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\boldsymbol{H} + 4\pi \boldsymbol{M}) - \frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}(\boldsymbol{H} + 4\pi \boldsymbol{M}). \tag{11}$$

Let us write (11) as the projection on the z axis; after integrating, we find that

$$H_z + 4\pi M_z = f_2(z) \tag{12}$$

where $f_2(z)$ is an arbitrary function of the z coordinate.

Comparing (10) and (12), we find that $f_1(t) = f_2(z) = \text{const}$. In the absence of the pulse we have that $M_z = M_0$, $M_x = M_y = 0$. Then $f_1 = f_2 = 4\pi M_0$ and, therefore,

$$H_z = 4\pi(M_0 - M_z). \tag{13}$$

After substituting (9) into (6) we find that

$$\partial M_\perp/\partial t = -\mathrm{i}(2\beta_0/\hbar)[(H_0 + H_z + \beta M_z)M_\perp - M_z H_\perp] \tag{14}$$

$$\partial M_z/\partial t = (2\beta_0/\hbar)(H_y M_x - H_x M_y) = (2\beta_0/\hbar)\,\mathrm{Im}(H_\perp M_\perp^*) \tag{15}$$

where $M_\perp = M_x + \mathrm{i}M_y$ and $H_\perp = H_x + \mathrm{i}H_y$.

Taking into account expression (13), we can rewrite equation (14) as

$$\partial M_\perp/\partial t = -\mathrm{i}\tilde{\omega}_\mathrm{H} M_\perp + \mathrm{i}\Psi M_z \tag{16}$$

where $\tilde{\omega}_\mathrm{H} = \omega_\mathrm{H}(1+\lambda)$, $\lambda = 8\pi\beta_0 M_0/(\hbar\omega_\mathrm{H})$ and

$$\Psi = 2\beta_0 \tilde{H}_\perp/\hbar \equiv 2\beta_0[H_\perp - (\beta - 4\pi)M_\perp]/\hbar. \tag{17}$$

Let us note that in the brackets on the right-hand side of equation (15) we can carry out the following replacement: $H_\perp \to H_\perp - (\beta - 4\pi)M_\perp \equiv \tilde{H}_\perp$. This replacement has no effect on the dynamical process, inasmuch as $M_\perp^* M_\perp$ is real. This is convenient for the further consideration in a formal manner. Then instead of (15) we will have:

$$\partial M_z/\partial t = \mathrm{Im}(\Psi M_\perp^*). \tag{18}$$

The equations (16) and (18) are the Bloch-like set of equations for the magnetization vector components.

From (16)–(18) it follows that the magnetization vector evolves under the effect of the transverse component of an effective transverse magnetic field $\boldsymbol{H}_\perp$. This field is the superposition of the transverse component of pulsed magnetic field $\boldsymbol{H}_\perp$ and of the magnetization field that is induced by the relative anisotropic interactions, $\beta \boldsymbol{M}_\perp$. The component $4\pi \boldsymbol{M}_\perp$ results from the contribution of the longitudinal pulse component H_z (see (13)).

The transverse projection (11), subject to (17), can be written as an equation for the complex function Ψ:

$$\frac{\partial^2\Psi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{2\beta_0\beta}{\hbar c^2}\frac{\partial^2 M_\perp}{\partial t^2} - \frac{2\beta_0}{\hbar}(\beta - 4\pi)\frac{\partial^2 M_\perp}{\partial z^2} + \frac{4\pi\sigma}{c^2}\frac{\partial}{\partial t}\left(\Psi + \frac{2\beta_0\beta}{\hbar}M_\perp\right). \tag{19}$$

The set of equations (16), (18) and (19) describes the magnetization dynamics of a ferromagnet and pulsed magnetic field in a self-consistent manner.

4. Solutions in the form of autowave dissipative structures

Executing the following substitution in (16) and (18)

$$\Psi = \mathrm{i}|\Psi|\exp(\mathrm{i}\varphi) \qquad M_\perp = S\exp(\mathrm{i}\varphi) \tag{20}$$

we obtain

$$\partial S/\partial t = -\mathrm{i}(\tilde{\omega}_\mathrm{H} + \partial\varphi/\partial t)S - |\Psi|M_z \tag{21}$$

$$\partial M_z/\partial t = |\Psi|\,\mathrm{Re}\,S^*. \tag{22}$$

Then, following Belenov *et al* (1988, 1991), Sazonov (1993) and Sazonov and Trifonov (1994), we assume that a pulse is too short if its duration is

$$1/\tau \gg \tilde{\omega}_\mathrm{H}, |\partial\varphi/\partial t|. \tag{23}$$

Under condition (23) a pulse interacts with a ferromagnet, including its very strong excitation, if the pulse parameters change rapidly in times ω_H^{-1}. The condition $1/\tau \gg |\partial\varphi/\partial t|$ makes it clear that the pulse polarization plane rotates far more slowly than the electron spin quantum transition, forming the magnetization field of a ferromagnet, occurs. Then on the right-hand side of (21) the first term can be ignored. Thereafter the variable S, as well as M_z and $|\Psi|$, become real, and we have the obvious solutions

$$M_z = M_0\cos\theta \tag{24}$$

$$S = -M_0\sin\theta \tag{25}$$

where

$$\theta = \int_{-\infty}^{t} |\Psi(z, t')|\,\mathrm{d}t'.$$

Using (16), (20) and (25), we find in the first approximation:

$$\partial M_\perp/\partial t = M_0\exp(\mathrm{i}\varphi)(\mathrm{i}\tilde{\omega}_0\sin\theta - F\cos\theta). \tag{26}$$

Solutions (24)–(26) generalize the corresponding solutions obtained for the case of optical pulses of linear polarization (Belenov *et al* 1988, 1991, Belenov and Nazarkin 1990).

Below we will assume that the pulse velocity is near to the speed of light. Therefore, on the right-hand side of equation (19) one can find terms of a higher order of smallness than on the left-hand side. This enables one to reduce the derivative order in (19). For this purpose we introduce the local time $T = t - z/c$ and the slow coordinate $\zeta = \mu z$ (Lamb 1980). Here μ is a small parameter corresponding to the ratio of the right-hand and left-hand sides of (19). Then we have

$$\begin{aligned} &\frac{\partial}{\partial t} = \frac{\partial}{\partial T} \qquad \frac{\partial}{\partial z} = -\frac{1}{c}\frac{\partial}{\partial T} + \mu\frac{\partial}{\partial\zeta} \\ &\frac{\partial^2}{\partial z^2} \simeq \frac{1}{c^2}\frac{\partial^2}{\partial T^2} - \frac{2\mu}{c}\frac{\partial^2}{\partial\zeta\,\partial T} = \frac{1}{c^2}\frac{\partial^2}{\partial T^2} - \frac{2}{c}\frac{\partial^2}{\partial z\,\partial T}. \end{aligned} \tag{27}$$

In the last expression (27) we ignored the term proportional to μ^2. Substituting (27) into (19), upon integrating over T, we obtain

$$\frac{\partial \Psi}{\partial z} = -\frac{2\pi\sigma}{c}\left(\Psi + \frac{2\beta_0\beta}{\hbar}M_\perp\right) - \frac{\beta_0}{\hbar c}\frac{\partial M_\perp}{\partial T} - \frac{2\beta_0}{\hbar}(\beta - 4\pi)\frac{\partial M_\perp}{\partial z}. \tag{28}$$

Substituting (20) and (26) into (28) and separating the real part from the imaginary part of the equation obtained, we find the following set of equations:

$$\frac{\partial^2\theta}{\partial z\,\partial T} + \frac{2\pi\sigma}{c}\frac{\partial\theta}{\partial T} + \frac{\beta_0}{\hbar c}M_0\left(\tilde{\omega}_0\sin\theta - 2c(\beta - 4\pi)\frac{\partial\varphi}{\partial z}\right)\sin\theta = 0 \tag{29}$$

$$\frac{\partial\theta}{\partial T}\frac{\partial\varphi}{\partial z} + \frac{\beta_0}{\hbar c}M_0\left[4\pi\sigma\beta\sin\theta + \left(\frac{\partial\theta}{\partial T} + 2c(\beta - 4\pi)\frac{\partial\theta}{\partial z}\right)\cos\theta\right] = 0. \tag{30}$$

Further we will cancel the set of non-linear equations (29) and (30). In an initial state let the magnetization vector be directed against the magnetic field:

$$M_0 = -2\beta_0 n \tag{31}$$

where n is the concentration of electron spins producing the ferromagnet.

We find the solutions for $\theta(z, T)$ as a running wave:

$$\theta = \theta(T - z/a) = \theta(t - z/v)$$

where $1/v = 1/c + 1/a$, a is a constant and v is the pulse velocity. Besides, we will employ the following *ansatz*:

$$\dot{\theta} = (1/\tau)\sin\theta. \tag{32}$$

Here the dot above θ designates the derivative with respect to $T - z/a$. Substituting (32) into (30) we find that

$$\partial\varphi/\partial z = (\Omega/c)\{4\pi\sigma\beta\tau - [(2c/a)(\beta - 4\pi) - 1]\cos\theta\} \tag{33}$$

where $\Omega = 2\beta_0^2 n/\hbar$.

Substituting (33) and (31) into (29), equating the coefficients of $\sin\theta$ and $\sin(2\theta)$ to zero after simple algebraic transformation, we find that

$$\frac{1}{\tau_\pm} = \Omega\left\{\frac{\tilde{\omega}_0}{4\pi\sigma} \pm \left[\left(\frac{\tilde{\omega}_0}{4\pi\sigma}\right)^2 - 4\beta(\beta - 4\pi)\right]^{1/2}\right\} \tag{34}$$

$$\frac{1}{v_\pm} = \frac{1}{c}\left(1 + \frac{2(\beta - 4\pi)\Omega^2}{\tau_\pm^{-2} + 4(\beta - 4\pi)^2\Omega^2}\right). \tag{35}$$

Integrating (32), using (35), (25), (24), (20), (17) and (13), we will obtain after going to the motionless framework:

$$H_\perp^\pm = [2\beta_0 n(\beta - 4\pi) + i\hbar/(2\beta_0\tau_\pm)]\exp(i\varphi)\,\mathrm{sech}[(t - z/v)/\tau_\pm] \tag{36}$$

$$H_\parallel^\pm \equiv H_0 + H_z^\pm = H_0 - 8\pi\beta_0 n\{1 + \tanh[(t - z/v)/\tau_\pm]\} \tag{37}$$

$$M_\perp^\pm = 2\beta_0 n\exp(i\varphi)\,\mathrm{sech}[(t - z/v)/\tau_\pm] \tag{38}$$

$$M_z^\pm = 2\beta_0 n\tanh[(t - z/v)/\tau_\pm] \tag{39}$$

$$\varphi_\pm = R_\pm z + \{1/[2(\beta - 4\pi)\Omega\tau_\pm]\}\log\{\cosh[(t - z/v)/\tau_\pm]\} \tag{40}$$

where

$$R_{\pm} = (4\pi\beta\sigma/c)\Omega\tau_{\pm}. \tag{41}$$

Obviously, from (3) it follows that $E_z = 0$. The two other components of the pulsed electric field can be found by using (2), (37) and (38).

Solutions (36)–(41) represent a dissipative autowave of the vector fields $\boldsymbol{H}$ (see figure 1) and $\boldsymbol{M}$. With regard to M_z and H_z the autowave is a running front, whereas with respect to $H_\perp$ and $M_\perp$ it is a running pulse. The pulses propagating in a medium of inverted spins take the energy accumulated in these spins. The amplification process is saturated by conduction electrons depriving a pulse of this energy, irreversibly losing it on collision with a crystal. Thus, a dissipative structure (36)–(41) can be formed.

Note that in a framework moving with the pulse velocity v we have the polarization plane rotation of transverse components of fields $\boldsymbol{H}$ and $\boldsymbol{M}$. This rotation is defined by the first term on the right-hand side of (40). The characteristic scale length at which the rotation angle is equal to 1 rad is determined by the value $R_{\pm}^{-1}$.

As $|\tilde{\Omega}| > 0$ and $\theta = |\Omega| = \tau^{-1}\sin\theta > 0$ $(0 < \theta < \pi)$, then we have that $\tilde{\omega}_{\mathrm{H}} > 0$. This inequality imposes the following restriction on the value of the Zeeman splitting frequency:

$$\omega_{\mathrm{H}} = g_{\parallel}\beta_0 H_0/\hbar > (16\pi/\hbar)\beta_0^2 n. \tag{42}$$

Putting $n \sim 10^{23}$ cm^{-3} we obtain that $\omega_{\mathrm{H}} > 5\times 10^{11}$ s^{-1}.

From (34) we obtain that $(\tilde{\omega}_{\mathrm{H}}/4\pi\sigma)^2 > 4\beta(\beta - 4\pi)$. It is clear that for this structure to exist one should meet the condition $v < c$. As follows from (35) this imposes the restriction on the magnetic anisotropy parameter: $\beta > 4\pi$. Summarizing we have the following conditions on the parameter β:

$$4\pi < \beta < 2\pi + [4\pi^2 + (\tilde{\omega}_{\mathrm{H}}/8\pi\sigma)^2]^{1/2}. \tag{43}$$

The conditions (42) and (43) are necessary for an electromagnetic autowave to be formed. Thus, a structure of the type (36)–(41) can be formed only in an easy-axis ferromagnet. Inequality $\beta > 4\pi$ can be fulfilled for any ferromagnetic monocrystals, e.g. for the cobalt monocrystal (Skrotskii and Kurbatov 1961).

The dynamic parameters of an autowave (velocity, amplitude, width) are defined by the medium parameters and are fully independent of initial conditions. This is understandable as the system under study is open and due to dissipation a pulse ‘forgets’ its initial conditions.

From (37) it follows that the value of longitudinal magnetic field changes irreversibly after pulse propagation. At $16\pi\beta_0 n/g_{\parallel} < H_0 < 16\pi\beta_0 n$ (see (37) and (42)) this field must have an opposite direction with regard to the direction of the initial field $\boldsymbol{H}_0$.

In further considerations the solution with V_+ and τ_+ will be described as an autowave of first type and the solution with V_- and τ_- as an autowave of second type.

Let $(\tilde{\omega}_{\mathrm{H}}/4\pi\sigma)^2 \gg 4\beta(\beta - 4\pi)$. Then from (34), (35) and (41) we obtain approximately

$$1/\tau_+ \simeq (\tilde{\omega}_{\mathrm{H}}/2\pi\sigma)\Omega \tag{44}$$

$$1/\tau_- \simeq (8\pi\sigma/\tilde{\omega}_{\mathrm{H}})\beta(\beta - 4\pi)\Omega \tag{45}$$

$$\frac{1}{v_+} \simeq \frac{1}{c}\left(1 + \frac{2(\beta - 4\pi)}{(\tilde{\omega}_{\mathrm{H}}/2\pi\sigma)^2 + 4(\beta - 4\pi)^2}\right) \tag{46}$$

$$\frac{1}{v_-} \simeq \frac{1}{c}\left(1 + \frac{1}{2(\beta - 4\pi)[1 + (4\pi\sigma\beta/\tilde{\omega}_{\mathrm{H}})^2]}\right) \tag{47}$$

$$R_+ \simeq (2\pi\sigma)^2 2\beta/c\tilde{\omega}_{\mathrm{H}} \tag{48}$$

$$R_- \simeq \tilde{\omega}_{\mathrm{H}}/[2c(\beta - 4\pi)]. \tag{49}$$

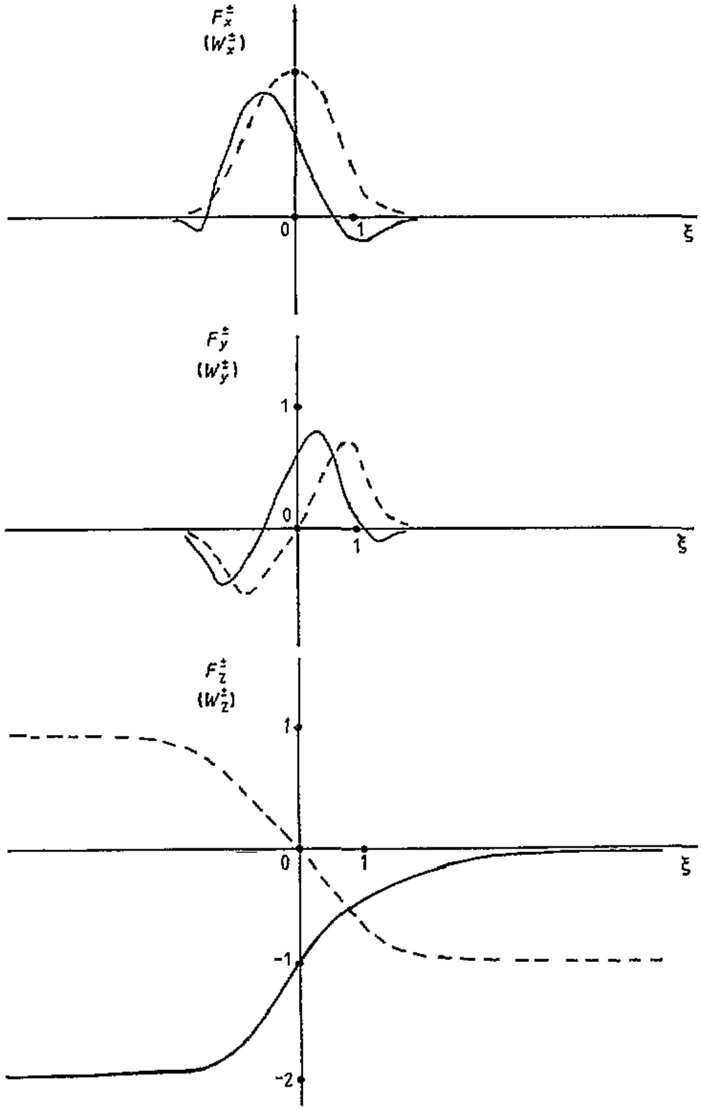

Figure 1. Instantaneous profiles of the autowave governed by equations (36)–(40) at $(4\pi\beta\sigma\Omega V_\pm\tau_\pm/c)t = 2\pi\ N(N = 0, \pm1, \pm2, \ldots)$. Here

$$F_{x,y}^\pm \equiv H_{x,y}^\pm/[4\beta_0^2 n^2(\beta - 4\pi)^2 + (\hbar/2\beta_0\tau_\pm)^2]^{1/2}$$

$$F_z^\pm \equiv H_z^\pm/(8\pi\beta_0 n) \qquad W_{x,y,z}^\pm \equiv M_{x,y,z}^\pm/(2\beta_0 n).$$

Full (broken) curves are the profiles of magnetic field (magnetization) components. The rotation phaseshift between transverse components of $\boldsymbol{H}$ and $\boldsymbol{M}$ is equal to $\tan^{-1}\{\hbar/[4\beta_0^2 n(\beta - 4\pi)\tau_\pm]\}$. At other moments of time the components $H_x^\pm$, $H_y^\pm$, $M_x^\pm$ and $M_y^\pm$ (but not $H_z^\pm$ and $M_z^\pm$) change their configurations in an accompanying framework due to the polarization plane rotation. However, in doing so, the area of the pulse localization determined by the scale $V_\pm\tau_\pm$ is retained.

At $n \sim 10^{23}$ cm^{-3}, $\beta \sim 10^2$, $2\pi\sigma \sim 10^8$ s^{-1} and $\tilde{\omega}_H \sim 10^{11}$ s^{-1}, we have the following values of pulse parameters: $\Omega \sim 10^{11}\ \mathrm{s}^{-1}$, $\tau_+ \sim 10^{-14}$ s, $\tau_- \sim 10^{-12}$ s, $R_+ \sim 10^{-3}\ \mathrm{cm}^{-1}$ and $R_- \sim 10^{-2}\ \mathrm{cm}^{-1}$. The condition $\tau_{\pm}^{-1} \gg |\partial\varphi/\partial t|$ is equivalent to the following inequality (see (40)):

$$2(\beta - 4\pi)\Omega\tau_{\pm} \gg 1. \tag{50}$$

Substituting these values of the pulse parameters into (50) we conclude that inequality (50) is fulfilled only for an autowave of second type, whereas for the pulse of first type this condition is not fulfilled. Therefore, in this case only a second-type autowave can be realized. Note that parameter R_+ decreases with increase of Zeeman splitting frequency ω_{H}; at the same time parameter R_- increases and practically does not depend on the electrical conductivity.

For the first- (second-) type autowave we have: $\hbar/(2\beta_0\tau_{+(-)}) \gg (\ll)\, 2\beta_0 n(\beta - 4\pi)$ (see (36)). Therefore, in the formation of the first- (second-) type autowave the non-linearity caused by the spin-wave (spin–spin or spin–orbit) interactions has a dominant role.

5. Concluding remarks

From (34) and (35) it follows that there are two solutions of (36)–(41) type. Probably this bistability is determined by the medium parameters and also by the initial conditions. At some initial conditions an autowave of first type can be formed; at the same time with other initial conditions we can obtain an autowave of second type. A similar situation occurs in the case of acoustic video pulses in a paramagnet lattice (Sazonov 1992b).

The electrical conductivity σ can be estimated by using the following expression:

$$4\pi\sigma \sim \omega_{\mathrm{L}}^2\tau_{\mathrm{e}}$$

where ω_{L} is the Langmuir frequency for the conduction electrons. One should bear in mind that for (6) to be valid the following condition must be met: $\tau_{\mathrm{e}} \ll \tau_{\pm}$. Having $\tau_- \sim 10^{-12}$ s, $\tau_{\mathrm{e}} \sim 10^{-13}$ s and $4\pi\sigma \sim 10^8\ \mathrm{s}^{-1}$, we find that $\omega_{\mathrm{L}} \sim 10^{11}\ \mathrm{s}^{-1}$. This value of Langmuir frequency corresponds to $n_{\mathrm{e}} \sim 10^{13}\ \mathrm{cm}^{-3}$, where n_{e} is the concentration of conduction electrons. This very low electron concentration corresponds to slowly conductive ferromagnets.

Acknowledgments

I would like to thank Dr Yu V Brezhnev for interesting discussions. It is my pleasure to thank Ms I V Vorobeikina for help in the typing of this paper. This work was carried out with the financial support of the Russian Basic Research Foundation (grant 93-02-15090).

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